Appendix 2 (as supplied by the authors): Conditional prediction of future FEV_1 given bassline FEV_1 values from the final model

In our model, observed FEV_1 is modeled as

$$FEV_t = \beta_0 + \beta.X + \beta_0'.t + \beta'.X.t \ + \beta_0''.t^2 + int * sign(t) + e,$$

Where β_0 (intercept) and β_0' (slope) are random, represented by a bivariate normal distribution whose parameters are directly estimated from the regression model,

$$(\beta_0, \beta_0') \sim BVN(mean = (\overline{\beta_0}\ ,\ \overline{\beta_0'},), covariance = \begin{bmatrix} Var(\beta_0) & Cov(\beta_0, \beta_0') \\ Cov(\beta_0, \beta_0') & Var(\beta_0') \end{bmatrix}).$$

Other coefficients are fixed. The int represents receiving intervention and is modeled only for follow-up FEV_1s (where $sign(t) = 1$), X is the set of covariates (i.e., baseline age, sex, weight, height, height squared, smoking status, O'connor slope, and interaction of baseline age and height squared), t represents time (year), and e is a normally distributed error term for the prediction.

This model connects any two FEV_1 values among patients as having bivariate normal distributions. A follow-up FEV_1 and baseline FEV_1 can therefore be jointly parameterized as:

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} \overline{\beta_0} + \beta.X + \ \beta'.X.t + \overline{\beta_0'}\,.t + \beta_0''.t^2 + int \\ \overline{\beta_0} + \beta.X \end{bmatrix},$$

$$cov = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} = \begin{bmatrix} Var(\beta_0) + 2.t.cov(\beta_0, \beta_0') + t^2.var(\beta_0') + var(e) & Var(\beta_0) + t.cov(\beta_0, \beta_0') \\ Var(\beta_0) + t.cov(\beta_0, \beta_0') & Var(\beta_0) + var(e) \end{bmatrix}.$$

Conditional on observed baseline FEV_1 having a value of Y_0, FEV_t gets an updated normal distribution with the following parameters:

$$\mu'(mean) = \mu_1 + \ \Sigma_{12}\ .\Sigma_{22}\ ^{-1}.(Y_0 - \mu_2),$$

$$v'(variance) = \Sigma_{11}\ - \ \Sigma_{12}\ .\Sigma_{22}\ ^{-1}.\Sigma_{21}\ \ (2).$$

For the incorporation of the previous history of FEV_1, the same line of reasoning is applied, noticing that any three FEV_1 values for the same individual have tri-variate normal distribution and the conditional mean and variance of follow-up FEV_1 given the baseline and previous FEV_1 can be obtained using the same matrix equations where μ_2, Σ_{12} , Σ_{21} , and Σ_{22} are updated as follows:

$$\mu_2 = \begin{bmatrix} \overline{\beta_0} + \beta.X \\ \overline{\beta_0} + \beta.X + \ \beta'.X.(-1) + \overline{\beta_0'}\,.(-1) + \beta_0''.(-1)^2 \end{bmatrix},$$

$$\Sigma_{12} = [Var(\beta_0) + t.cov(\beta_0, \beta_0') \quad Var(\beta_0) - cov(\beta_0, \beta_0') + t.cov(\beta_0, \beta_0') - t.var(\beta_0')],$$

$$\Sigma_{21} = \begin{bmatrix} Var(\beta_0) + t.cov(\beta_0, \beta_0') \\ Var(\beta_0) - cov(\beta_0, \beta_0') + t.cov(\beta_0, \beta_0') - t.var(\beta_0') \end{bmatrix},$$

$$\Sigma_{22} = \begin{bmatrix} var(\beta_0) + var(e) & var(\beta_0) - cov(\beta_0, \beta_0') \\ var(\beta_0) - cov(\beta_0, \beta_0') & Var(\beta_0) - 2.cov(\beta_0, \beta_0') + var(\beta_0') + var(e) \end{bmatrix}.$$

Appendix to: Zafari Z, Sin DD, Postma DS, et al. Individualized prediction of lung function decline in chronic obstructive pulmonary disease.
CMAJ 2016. DOI:10.1503/cmaj.151483.