Appendix 5: Fractional polynomial transformations

Fractional polynomial transformations allow modelling of complex non-linear relationships between an outcome and a continuous predictor. In this analysis, the best fitting curves describing the relationships between pain or disability scores and weeks required two coefficients for time to be added to the equation of the fitted line. All combinations of -2 (1/weeks^2), -1 (1/weeks), -0.5 (1/sqrt(weeks), 0 (ln(weeks)), 0.5 (sqrt(weeks)), 1 (weeks), 2 (weeks^2) and 3 (weeks ^3) were considered. In all cases weeks were scaled by adding 1 so that baseline (zero) weeks could be modelled, because many of these transformations are undefined for 0.

As an example, consider the fitted line equation for disability score (acute cohort).

Pooled disability score = $17.17116 + 26.1946*(\text{Week} + 1)^{(-2)} + 118.6091*(\text{Week} + 1)^{(-2)} * \ln(\text{Week} + 1)$.

The FP transformation here is -2 -2.

So for 6 weeks, the value $1/(6 +1)^2$ is substituted in for $(\text{Week} + 1)^{(-2)}$. As -2 was repeated, the natural log of week is included as part of the transformation for the second coefficient above. This is done whenever the FP power repeats.

For fractional polynomial transformations, raising a number to the power of 0 equals the natural logarithm of that number. Therefore, the natural log transformations used in this study are FP transformations of 0.